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Aftermarket power and foremarket competition ☆

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ABSTRACT

I revisit the issue of aftermarkets by developing an infinite period model with overlapping consumers. If the aftermarket is characterized by constant returns to scale, then social surplus and consumer surplus are invariant with respect to aftermarket power. Under increasing returns to scale, however, greater aftermarket power leads to: greater concentration in the foremarket; higher barriers to entry; higher social surplus; and possibly higher consumer surplus.

1. Introduction

Many consumers complain that they pay too much for printer toners. But the same consumers are also happy to purchase printers at fairly low prices. To some extent, lower printer prices compensate for higher toner prices. Or do they?

The printer–toner example is one of many instances of industries characterized by a foremarket that is complemented by one or several aftermarkets. Typically, the foremarket corresponds to a durable good, whereas the aftermarkets correspond to non-durable products or services. Other than printers, examples include cameras and film, photocopiers and repair service, videogame consoles and games.[1]

In these industries, an interesting policy question is how to treat seller power in the aftermarket. An old argument (associated to the Chicago school) states that a seller can only have so much market power, and that an increase in aftermarket power is compensated by an equal decrease in power in the foremarket: the price of blades may be very high, but razor holders are very cheap.[2] Some authors argue that the conditions for such an equivalence result are very stringent. For example, Borenstein et al. (1995) claim that "economic theory does not support the argument that strong primary market competition will discipline aftermarket behavior, even without market imperfections" (p. 459). Other authors, while recognizing the welfare reducing effects of market power, suggest that these are rather small in magnitude. For example, Shapiro (1995) concludes that "significant or long-lived consumer injury based on monopolized aftermarkets is likely to be rare, especially if equipment markets are competitive" (p. 485).

In addition to market power, efficiency considerations may also play an important role. For example, where there is a risk of shared liability between an equipment manufacturer and a third party service provider, aftermarket power may be a "necessary evil." As another example, having the same seller supply both the basic product and the aftermarket product may also allow for welfare enhancing price discrimination.

The current US antitrust treatment of aftermarket power is largely based on the *Kodak* case. Kodak refused to sell spare parts to third parties offering after-sales photocopier services, thus effectively monopolizing an important aftermarket. In its defense, Kodak argued that, although it effectively monopolized the aftermarket, its share of the foremarket was only 2%. In its 1992 decision, the US Supreme Court decided held that lack of market power in the primary equipment market does not necessarily preclude antitrust liability for exclusionary conduct in derivative aftermarkets.[3]

Unlike the US Supreme Court, the European Commission (EC) and the European Court of Justice (ECJ), deciding on the *Kyocera* case, held

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[1] A related, but different, setting consists of secondary markets where the initial good may be resold. See for example Hendel and Lizzeri (1999).

[2] See Picker (2011) for a critical view on the common wisdom regarding razors and razor blades pricing.

[3] *Eastman Kodak Company v. Image Technical Services, Inc.*, 504 U.S. 451 (1992). Justice Scalia dissented, arguing that Kodak lacked power in the equipment market.

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that consumers are well informed and take into account aftermarket prices when choosing a certain piece of equipment. Since there is vigorous competition in the primary market for printers, the EC argued, Kyocera was not dominant in the market for printer consumables (toners).[4]

Although the *Kodak* and *Kyocera* decisions differ in several ways, both recognize the importance of economic analysis, in particular the simultaneous consideration of power in the foremarket and in the aftermarket.

In this paper, I revisit the relation between aftermarket power and foremarket competition. The novel element of my analysis is to consider a dynamic (infinite period) model with increasing returns to scale in the aftermarket (which may result from economies of scale, indirect network effects, or other causes). I assume consumers' lives overlap with one another. In each period, one consumer is born and joins one of the existing installed bases; next, aftermarket payoffs are received by sellers and consumers; and finally, one consumer dies. I derive the unique symmetric Markov equilibrium of this game and the resulting stationary distribution over states (which correspond to each firm's installed base).

I show that increasing returns in the aftermarket induce increasing dominance in the foremarket; that is, under increasing returns a large firm is more likely to capture a new consumer than a small firm. Moreover, an increase in aftermarket power increases the extent of increasing dominance. This in turn has several implications. First, aftermarket power implies a stationary distribution with greater weight on asymmetric states. Second, social welfare is greater with aftermarket power (basically because social welfare is higher at asymmetric states). Third, the value of a small firm (a firm with no installed base) is lower when there is aftermarket power. Fourth, because the difference in value between large firms and small firms widens, firms compete more aggressively to attract new customers when there is aftermarket power. And finally, because of more aggressive price competition, consumer welfare may be greater when there is aftermarket power.

Intuitively, my results are related to two important features of dynamic price competition. The first one is the *efficiency* or *joint profit* effect.[5] The idea is that a large firm has more to lose from decreasing its market share than a small firm has to gain from increasing its market share. This induces the large firm to be relatively more aggressive and makes the next sale with greater probability than the small firm: increasing dominance. In my model, I show that aftermarket power increases the stakes that firms compete for; and this in turn increases the extent of increasing dominance.

The second feature is what we might call the *Bertrand supertrap* effect.[6] Consider a symmetric bidding game, where the winner receives w and the loser gets l. Equilibrium bids are given by $w–l$; it follows that each player's equilibrium payoff is given by l: if you win, you get w, but you also have to pay $w–l$. In the present context, I show that aftermarket power, while increasing future profits, makes firms so much more competitive that, starting from a symmetric state, firms are worse off, whereas consumers are better off. In other words, in terms of future value a large firm is better off with aftermarket power, but a small firm is worse off; and the latter is what matters in terms of present value.

In terms of competition policy, my paper makes two points. First, given a set of firms and product offerings, consumers need not be harmed by aftermarket power. In fact, to the extent that there are increasing returns in the aftermarket and the foremarket is competitive, consumers can be strictly better off in the presence of aftermarket power. (Several authors have argued that aftermarket power may be welfare increasing, but for different reasons than the one I consider; more on this below.) Second, increases in aftermarket power have important implications for market share dynamics. On average, foremarket concentration increases; and the barriers to entry of new firms increase as well. Taken together, these two points suggest that aftermarket power raises concerns from a consumer welfare point of view, but not for the reasons typically considered in the literature.

Prior literature on aftermarkets can be divided into two groups. (In both cases, the approach is essentially theoretical, although the motivation is grounded on actual cases.) One first strand looks at the balance between aftermarket power and foremarket competition. The early development of this literature is aptly summarized in Shapiro (1995), who acknowledges the potential for aftermarket power to reduce consumer welfare but estimates the impact not to be too significant. More recently, Fong (2008)shows that aftermarket power may enhance collusion. Zēgners and Kretschmer (2014), in turn, show that aftermarket power leads to lower prices in the foremarket, which in turn may inefficiently attract consumers whose valuation is lower than cost.

A second strand of the literature studies efficiency defenses of aftermarket power. For example, Chen and Ross (1993) argue that a seller may use the aftermarket as a "metering device to discriminate between high-intensity, high-value users and low-intensity, low-value users" (p. 139); whereas Carlton and Waldman (2010) show that "behaviors that hurt competition in aftermarkets can … be efficient responses to potential inefficiencies that can arise in aftermarkets."[7]

My paper can be seen as a contribution to both strands of the literature. First, it confirms the well-known idea that increases in aftermarket power are compensated by increases in foremarket competition, with the important qualification that, under increasing returns, the increase in competition in the foremarket exceeds the increase in power in the aftermarket. Second, I add a novel reason why aftermarket power may lead to efficiency gains, namely a better exploitation of increasing returns to scale — so much so that even consumers may benefit from aftermarket power.

As mentioned earlier, from a methodological point of view an important difference with respect to the previous literature is the development of an infinite period dynamic model where the state space is given by the installed base of each firm. In this sense, the paper is closely related to Cabral (2011), who studies dynamic price competition with network effects.[8] The present paper differs from Cabral (2011) in several ways. First, it puts more structure into the model so as to analyze the issue of aftermarket power explicitly. In particular, the central results in the present paper — that aftermarket power increases social welfare and may increase consumer welfare as well — are not present in Cabral (2011). Second, by considering specific functional forms, the present paper derives analytical results for ranges of parameter values where Cabral (2011) only obtained numerical results. In particular, the results regarding increasing dominance (bigger firms are more likely to make the next sale than smaller firms) are derived analytically for all parameter values, whereas Cabral (2011) only develops analytical results for limit values.

The paper is organized as follows. In Section 2, I introduce my dynamic model of foremarket and aftermarket competition. In Section 3, I consider the benchmark case of constant returns to scale and show that the one-monopoly-rent principle holds. In Section 4, I consider the case of increasing returns to scale in the aftermarket and two possible aftermarket configurations: perfect competition and monopoly. I prove that aftermarket power increases the degree of increasing dominance. Section 5 derives two implications of this result, one regarding long-run market shares, one regarding barriers to entry. Section 6 deals with social and consumer welfare. Finally, Section 7 concludes the paper.

4 *Pelikan/Kyocera*, (1996) 17 ECLR R-57.

5 See Gilbert et al. (1982), Budd et al. (1993), Cabral and Riordan (1994), Athey and Schmutzler (2001).

6 See Cabral and Riordan (1994), Cabral and Villas-Boas (2005).

7 See Chen et al. (1998) for a review on the economics and legal literature on aftermarkets.

8 See also Laussel and Resende (2014).

2. Model[9]

Consider an industry with two sellers and an infinite series of overlapping consumers. In each period, one consumer is born and endowed with preferences for seller i's basic good. Sellers simultaneously set prices p_i for that good and the consumer chooses one of the sellers. Next, all consumers, old and new, purchase a complementary good in the aftermarket. I will not model in detail the nature of aftermarket competition. Rather, I assume that firm i receives a profit π_i, whereas a consumer attached to firm i earns a surplus λ_i (all consumers value the aftermarket good equally). Finally, at the end of the period one of the consumers dies, each with equal probability.

One possible interpretation of my consumer birth and death process is that there is a fixed number of consumers who only make durable goods-purchase decisions occasionally. For example, most consumers don't think about changing their printer on a daily basis. If the printer breaks down, or if it requires a new toner, or if the consumer moves to a different job, or if the consumer is shown another printer that works a lot better — then the consumer re-considers his choice of a printer. In terms of my model, I assume the above events are exogenous and denote them by consumer "death." In other words, when the consumer is subject to a shock that leads him to re-consider what durable good to use, I assume the consumer "dies" and is immediately "re-born", at which moment he makes a new durable-good choice.

Notice that I implicitly assume that, in between durable-good choices, the consumer stays with the same durable good. In other words, I assume that "death" is exogenous. In fact, my model may be interpreted as a model where consumers have a stochastic switching cost, the value of which is either infinite (while the consumer is "alive") or zero (when the consumer "dies"). Admittedly, this is a somewhat extreme assumption, but in many ways a more realistic one than the "standard" switching costs model where the consumer makes a fully-informed choice in each period and faces a constant switching cost.

Throughout the paper, I assume that, in each period, there are 3 consumers in the aftermarket. This implies that, at the beginning of the period, there are 2 old consumers. Given symmetry, we have two possibilities: either both firms have the same installed base (1 consumer each) or one of the firms has a large installed base (2 consumers) whereas the other firm has a zero installed base.

I will be looking at symmetric Markov equilibria, where the state of the game is given by the firms' installed bases.[10] For simplicity, if with some abuse of notation, I will denote by i the size of firm i's installed base. At the beginning of each period, we thus have $i+j=2$.

My model is obviously very stylized, and some of the assumptions rather stark. The assumption that there are only 3 consumers allows me to derive close-form analytical expressions of the equilibrium variables.[11] The number 3 is the lowest that allows me to distinguish between symmetric and asymmetric states, a necessary feature for the main results of the paper.[12]

I next study in greater detail the consumers' and the firms' choice problems.

2.1. Consumer choice

A newborn consumer is endowed with valuations ζ_i for firm i's basic good. I assume the outside option is worth $-\infty$, that is, a newborn consumer always chooses one of the firms.[13] Given this assumption, the difference in consumer valuations, $\xi_i \equiv \zeta_i - \zeta_j$, is a sufficient statistic of consumer preferences. I assume that $\xi \sim U[-\frac{1}{2}, \frac{1}{2}]$.[14] Consider a new consumer's decision. In state i, the indifferent consumer has $\xi_i = x_i$, where the latter is given by

$$x_i - p_i + u_{i+1} = -p_j + u_{j+1} \tag{1}$$

where p_i is firm i's price and u_i is the consumer's aftermarket value function, that is, the discounted value of the stream of payoff λ_i received while the consumer is alive (thus excluding both ζ_i and the price paid for the basic good). Specifically, in each period that a consumer is alive he receives an aftermarket payoff λ_i, where i is the size of the installed-base *in that period*.

The above problem looks very much like a Hotelling consumer decision (with firms located at $-\frac{1}{2}$ and $\frac{1}{2}$ and unit transport cost), except for the fact that u_{i+1} and u_{j+1} and endogenous values.

Firm i's demand is the probability of attracting the new consumer to its installed base. Since ξ_i is uniformly distributed in $[-\frac{1}{2}, \frac{1}{2}]$, we have $F(\xi_i) = \frac{1}{2} + \xi_i$. Therefore, the probability that firm i attracts a new consumer to its installed base, q_i, is given by

$$\begin{aligned} q_i \;=\; \mathcal{P}(\xi_i > x_i) = 1 - F(x_i) \;&=\; \frac{1}{2} - x_i = \\ &= \frac{1}{2} - \Big(\big(p_i - p_j\big) - \big(u_{i+1} - u_{j+1}\big)\Big) \end{aligned} \tag{2}$$

where the last equality follows from Eq. (1). Finally, the consumer value functions are given by

$$u_i = \lambda_i + \delta\left(\frac{j}{3}\, q_i\, u_{i+1} + \left(\frac{j}{3}\, q_{j-1} + \frac{i-1}{3}\, q_{i-1}\right) u_i + \frac{i-1}{3}\, q_j\, u_{i-1}\right) \tag{3}$$

$i=1,2,3$, $j=3-i$. In words, a consumer who is attached to an installed base of size i receives λ_i in the current period. Beginning next period, four things may happen: a consumer from installed base i dies and the new consumer joins installed base j, in which case continuation value is u_{i-1}; a consumer from installed base j dies and the new consumer joins installed base i, in which case continuation value is u_{i+1}; and two events where death and birth take place in the same installed base, in which case continuation payoff is u_i.

2.2. Firm's pricing decision

Assuming for simplicity zero production cost, firm i's value function is given by

$$\begin{aligned} v_i = q_i&\left(p_i + \pi_{i+1} + \delta\, \frac{j}{3}\, v_{i+1} + \delta\, \frac{i+1}{3}\, v_i\right) \\ &+ (1-q_i)\left(\pi_i + \delta\, \frac{j+1}{3}\, v_i + \delta\, \frac{i}{3}\, v_{i-1}\right) \end{aligned} \tag{4}$$

where $i=0,1,2$ and $j=2-i$. With probability q_i, firm i attracts the new consumer and receives p_i. This moves the aftermarket state to $i+1$, yielding a period payoff of π_{i+1}; following that, with probability $(i+1)/3$ firm i loses a consumer, in which case the state reverts back to i, whereas with probability $j/3$ firm j loses a consumer, in which case the

[9] Cabral (2011) presents a more general framework of dynamic price competition that includes the present model as a particular case. The cost of a general framework is that few analytical results are possible. By contrast, in the present context I am able to derive a closed-form analytical solution.

[10] I can show that, under certain conditions, model symmetry implies equilibrium symmetry. However, in what follows I will assume symmetry of equilibria. Notice however that symmetry of equilibrium does not imply symmetry of outcome. In fact, since the model is stochastic, the outcome is asymmetric with positive probability.

[11] In Cabral (2011) I present a more general framework but many results are obtained numerically.

[12] Numerical computations suggest that Propositions 1–4 generalize for higher value of n. I conjecture the same is true for Propositions 5 and 6.

[13] This is not a knife-edged assumption, that is, I could assume that the outside option is exercised with a low probability. However, if that were to happen with probability significantly greater than zero then some of my results would not hold.

[14] Similar to the number of consumers, this assumption is not essential but considerably simplifies the analysis.

state stays at $i+1$. With probability q_j, the rival firm makes the current sale. Firm i gets no revenues in the primary market. In the aftermarket, it gets π_i in the current period; following that, with probability $i/3$ network i loses a consumer, in which case the state drops to $i-1$, whereas with probability $(j+1)/3$ network j loses a consumer, in which case the state reverts back to i.

Eq. (4) leads to the following first-order conditions for firm value maximization:

$$q_i + \frac{\partial q_i}{\partial p_i}\left(p_i + \pi_{i+1} - \pi_i + \delta\ \frac{j}{3}\ v_{i+1} + \delta\ \frac{i+1}{3}\ v_i - \delta\ \frac{j+1}{3}\ v_i - \delta\ \frac{i}{3}\ v_{i-1}\right) = 0$$

or simply

$$p_i = q_i - (\pi_{i+1} - \pi_i) - \delta\left(\frac{j}{3}\ v_{i+1} + \frac{i-j}{3}\ v_i - \frac{i}{3}\ v_{i-1}\right). \quad (5)$$

Finally, substituting Eq. (5) into Eq. (4) and simplifying, we get

$$v_i = q_i^2 + \pi_i + \delta\left(\frac{j+1}{3}\ v_i + \frac{i}{3}\ v_{i-1}\right). \quad (6)$$

2.3. Equilibrium

A Symmetric Markov Nash equilibrium is a set of prices p_i and demands q_i for the basic good ($i = 0, 1, 2$), as well as a set of consumer value functions u_i ($i = 1, 2, 3$) and firm value functions v_i ($i = 0, 1, 2$), that satisfy Eqs. (2) and (5) (quantities and prices, respectively), (3) and (6) (consumer and firm value functions, respectively).

To help the reader navigate through a wide sea of notation, I denote endogenous variables with letter from the Roman alphabet: p, q, u, v; and exogenous parameters with letters from the Greek alphabet: π, λ. I next put a little more structure into these exogenous parameters.

2.4. Aftermarket conditions

In order to highlight the effects of market power as a transfer from buyer to seller, I assume that the aftermarket value created at each state is independent of seller power. Specifically, when one firm has an installed base of size i and its rival an installed base of size $j = 3 - i$, then total aftermarket welfare is given by

$$V_i \equiv \pi_i + \pi_j + i\ \lambda_i + j\ \lambda_j. \quad (7)$$

My assumption is that, as aftermarket power conditions change, V_i remains constant. In other words, aftermarket power is simply a transfer from consumer surplus (λ_i) to firm profits (π_i). There are reasons for total surplus to be decreasing in market power (the usual Harberger triangle) or increasing in market power (see, for example, Carlton and Waldman, 2010). My assumption is intended to focus on the effects of dynamic competition on consumer welfare and social welfare.

3. Constant returns to scale

In this section, I consider a benchmark case that essentially corresponds to results previously derived in the literature on aftermarkets. Suppose that

$$\begin{aligned} \lambda_i &= \lambda \\ \pi_i &= i\ \pi \end{aligned} \quad (8)$$

that is, we have constant returns to scale: a consumer's utility from using firm i's good is independent of firm i's size, and firm i's profit is proportional to size. My main result in this section is one of irrelevance: market share dynamics and consumer welfare are invariant with respect to aftermarket power.

Proposition 1. *Under constant returns to scale, equilibrium price and demand are constant across states. Moreover, consumer welfare is invariant with respect to aftermarket power.*

A complete proof of this and the next results may be found in the Appendix A sketch of the proof is as follows. Suppose firm value is proportional to installed base (the proof derives this result rather than assume it). Then the first order condition (5) shows that the second and third terms are independent of i: the "prize" from capturing a new consumer, both in terms of the current period's profits and in terms of future profits, is the same for small and large firms. Since consumers do not care about firm size (they always get λ in the aftermarket), it follows that equilibrium price in the foremarket is independent of firm size.

Given that price in the foremarket is independent of firm size, the only factor of variation in the value function is aftermarket profits. But then the increased discounted value of aftermarket profits is exactly competed away by pricing in the foremarket. In other words, the increase in firm profits from greater aftermarket power implies a higher "prize" for the firm that makes the current sale; and this prize is translated into lower prices in the foremarket by the same amount. It follows that the current newborn consumer is indifferent with respect to the degree of market power.

To summarize, under constant returns to scale in the aftermarket consumers are indifferent to the degree of aftermarket power. This corresponds to the well-known "Chicago School" result regarding aftermarket power: if the foremarket is competitive, then aftermarket power does not harm consumers. In this sense, my contribution is to show that the argument, usually cast in the context of a two-period model, extends to the case of an infinite-period model.

In the next sections, I compare this benchmark against the case of increasing returns to scale in the aftermarket. I show that, first, the dynamics are no longer trivial; and second, consumer and social welfare vary with aftermarket power in a nontrivial way.

4. Increasing dominance

In this and in the following sections, I consider the possibility of increasing returns to scale in the aftermarket, that is, the possibility that total surplus increases more than proportionately with the size of the installed base. There are several instances when this is a reasonable assumption. For example, suppose that in each period the seller makes an investment which increases the value of the aftermarket good or service, and suppose the cost of such investment is a function of the quality increase but not of the number of consumers. Then, the greater the number of consumers, the greater the marginal gain from investment, and the greater the total value generated in the aftermarket. A second source of increasing returns is network effects. For example, videogame players get a greater value out of a game to the extent that they can play it with other players, and so consumer surplus is likely to be increasing in the size of the installed base.

Regarding aftermarket power, I consider two extreme cases: Case *C*, when the aftermarket is competitive, and Case *M*, when the aftermarket is monopolized. Profit and consumer surplus in each case are given in Table 1. In the competition scenario, seller profits are zero, whereas

Table 1
Aftermarket conditions: firm profit, π_i, and consumer surplus, λ_i, as a function of installed base, i, under two possible cases.

	π_i	λ_i
C: Aftermarket competition	0	$\omega + i\ \phi$
M: Aftermarket monopoly	$i\ (\omega + i\ \phi)$	0

each consumer receives a surplus that is linear and increasing in installed base size: $\omega + i\,\phi$. In the monopoly scenario, a seller with an installed base i earns $\omega + i\,\phi$ per consumer, yielding a total of $i\,\omega + i^2\,\phi$, whereas consumers receive a surplus of zero. Note that $V_i \equiv \pi_i + \pi_j + i\,\lambda_i + j\,\lambda_j$ is the same in cases C and M.

As a concrete example corresponding to this functional form, consider the case when the foremarket corresponds to a video-game console, whereas the aftermarket corresponds to video-games. Each consumer gets a "stand-alone" utility ω from using the console. Moreover, with probability α a consumer meets another consumer; and, if the other consumer owns the same console, a benefit β is derived from that meeting (either because they share games or share experiences from playing games). Regarding competition, we have two options. Under monopoly, n video games are produced and sold for a price equal to the consumer's valuation. Under competition, every game is offered both by the firm and by a third party; the degree of differentiation between each game and its clone is sufficiently small that price competition implies zero profits. Finally, if we make $\phi = \alpha\,\beta$ then we get the payoffs described in the preceding paragraph and summarized in Table 1.[15]

My main result in this section is that increasing returns lead to increasing dominance, the property whereby firms with larger installed bases are more likely to attract new customers. Moreover, aftermarket power increases the degree of increasing dominance.

Let q_i^k be the probability that a seller with installed base i ($i = 0, 1, 2$) attracts the newborn consumer, assuming aftermarket conditions k ($k = C, M$). Notice that market shares must add up to 1, that is, $q_i^k + q_j^k = 1$. Given symmetry, we then have $q_1^k = \frac{1}{2}$. It follows that q_2^k (or alternatively $q_0^k = 1 - q_2^k$) is a sufficient statistic of market share dynamics.

Proposition 2. *The large seller is more likely to attract a new consumer, especially if sellers have aftermarket power:* $\frac{1}{2} < q_2^C < q_2^M$.

The Proof of Proposition 2 is not particularly simple or elegant, but the result is fairly intuitive. Specifically, the intuition for $q_2^k > \frac{1}{2}$ is that increasing returns in the aftermarket imply an *efficiency* or *joint profit* effect.[16] The idea is that firm value is a convex function of installed base size. This implies that a firm with an installed base of 2 has more to lose from dropping to 1 than a firm with an installed base of 0 has to gain from reaching 1. As a result, the large firm prices more aggressively and sells with greater probability. The intuition for $q_2^M > q_2^C$ is that an increase in aftermarket power increases the sellers' stakes in the aftermarket, that is, *magnifies the size of the efficiency effect*. This in turn results in a greater gap between the leader's and the follower's probability of attracting the newborn consumer.

5. Market concentration and barriers to entry

In this section, I derive two fairly straightforward implications of Proposition 2, both relating to the foremarket: one regarding market concentration and one regarding barriers to entry.

Let μ_i^k be the stationary probability of being in state i ($i = 0, 1, 2$) given aftermarket conditions k ($k = C, M$). Symmetry implies that $\mu_0 = \mu_2$: whenever a firm is in state 0 there must be a firm in state 2. Therefore, μ_1^k provides a sufficient statistic for the degree of foremarket concentration: the greater μ_1^k is, the longer the system spends at the symmetric state, that is, the less concentrated the foremarket is.

Proposition 3. *Aftermarket power implies foremarket concentration:* $\mu_1^M < \mu_1^C$.

The idea is simple: death rates are independent of aftermarket conditions, whereas birth rates for the large firm are higher under aftermarket power (by Proposition 2). Together, these facts imply that the stationary distribution places greater weight on asymmetric states the greater the degree of aftermarket power.

Is an industry more attractive if sellers have aftermarket power? One might be tempted to say yes: more rents create a better prospect for an entrant. However, one must take into account the effect that aftermarket power has on foremarket competition. In fact, for an entrant — that is, a firm that starts with an installed base of zero — all of the potential benefits from aftermarket power — and more — are competed away. In the end, an entrant is strictly worse off when there is aftermarket power. Let v_i^k be the value of a firm with installed base i given aftermarket conditions k ($k = C, M$). We then have the following result:

Proposition 4. *The value of a firm with no installed base is lower if sellers have aftermarket power:* $v_0^M < v_0^C$.

Proposition 4 follows naturally from Proposition 2. Increasing returns in the aftermarket make firms more aggressive, especially large firms. This hurts small firms: while market power increases expected aftermarket profits, this gain is more than compensated by the loss from the rival's lower prices in the foremarket.

A potential entrant compares the cost of entry to the expected benefit upon entry. Since an entrant starts with an installed base of zero, the expected benefit upon entry is given by v_0^k. For this reason, we may say that aftermarket power increases the size of the barriers to entry in the foremarket.

6. Social welfare and consumer welfare

Increasing returns to scale create a situation of natural monopoly: social welfare is greater the more concentrated markets are. As a result, market forces that imply greater concentration also increase welfare.

Proposition 5. *Social welfare is strictly greater if sellers have aftermarket power.*

Proposition 3 implies that, under aftermarket power, asymmetric installed bases are more likely. As mentioned above, this in turn implies greater total welfare. The Proof of Proposition 5 is not as trivial as it might seem because there is a countervailing effect on social welfare. To the extent that q_i is different from $\frac{1}{2}$, consumer "transportation" costs are greater than the minimum transportation costs. In other words, while the aftermarket component of social welfare is greater under aftermarket power, the primary market component is lower. In the proof, I show that the latter effect is dominated by the former.

The inequality in Proposition 5 is strict. This implies that I can perturb my assumption of constant total aftermarket surplus in each state i. Suppose that total aftermarket surplus is ϵ higher in Case C than in Case M. If I make ϵ small enough (a tiny Harberger triangle), then I can find an open set of parameter values such that (a) in each state, social welfare is *lower* when there is aftermarket power; (b) in the steady state, social welfare is *greater* when there is aftermarket power. The justification for this apparently contradictory statement is that aftermarket power, while leading to a tiny loss in total surplus in each state, leads to a reallocation of steady state probabilities that places greater weight in states with strictly higher total surplus (and by more than ϵ).

I finally turn to one of my main results: the effect of aftermarket power on consumer welfare. Much of the previous literature on aftermarkets attempted to establish whether the injury to consumers resulting from aftermarket power is or is not significant. By contrast, I show that aftermarket power may actually *increase* consumer welfare.

Proposition 6. *There exist* ϕ', δ' *such that, if* $\phi < \phi'$ *and* $\delta > \delta'$ *then consumers are strictly better off with aftermarket power.*

[15] Empirical evidence of (indirect) network effects on video games may be found in Clements and Ohashi (2005), Claussen et al. (2012). See also Goolsbee and Klenow (2002).

[16] See Gilbert et al. (1982), Budd et al. (1993), Cabral and Riordan (1994), Athey and Schmutzler (2001).

Table 2
Approximate values of equilibrium variables when $\delta = 1$ and ϕ is small.

	Aftermarket conditions	
	Competition	Monopoly
q_2	$\frac{1}{2}+\phi$	$\frac{1}{2}+2\ \phi$
p_0	$\frac{1}{2}-3\ \phi$	$\frac{1}{2}-9\ \phi$
p_1	$\frac{1}{2}-2\ \phi$	$\frac{1}{2}-10\ \phi$
$\underline{p_2}$	$\frac{1}{2}-\phi$	$\frac{1}{2}-11\ \phi$
$\sum\lambda$	$6\ \phi$	0
Consumer welfare	$\zeta'-\frac{1}{2}+8\ \phi$	$\zeta'-\frac{1}{2}+10\ \phi$

Table 2 may be useful in understanding the effects of market power on equilibrium values, in particular the level of consumer welfare. If $\delta = 1$ and $\phi = 0$, then consumer welfare is the same under aftermarket competition or monopoly (by Proposition 1). For $\delta = 1$ and small values of ϕ, I can approximate the values of the various endogenous variables by a linear expansion around $\phi = 0$.

First notice that q_2 is greater under aftermarket power. This is consistent with Proposition 2. The idea is that, under market power, large firms have more to lose from not attracting a newborn consumer than small firms have to gain from attracting that same consumer. This leads large firms to price more aggressively and newborn consumers to choose large firms more likely.

Aftermarket power has two important effects on firm pricing in the foremarket. First, prices are lower. The idea is that aftermarket power increases the prize from capturing an extra consumer, and foremarket prices move accordingly. Second, whereas under aftermarket competition prices are increasing in the size of the installed base, under aftermarket monopoly prices are decreasing in the size of the installed base. The reason is that, under dynamic competition, there are two forces determining optimal price, which we may refer to as the *harvesting* effect and the *investment* effect (cf Farrell and Klemperer, 2007). The idea of the harvesting effect is that, to the extent a larger firm offers a better good in the eyes of the consumer, such firm prices higher accordingly. This is the main effect at work when the aftermarket is competitive. The idea of the investment effect is that, to the extent the value function is convex, a larger firm has more to gain from attracting the newborn consumer. This effect dominates when the aftermarket is monopolized.

Finally, we come to consumer welfare. There are two components to take into account: the aftermarket component and the foremarket component. Under aftermarket competition, consumers expect a positive surplus in the aftermarket. On average, in the steady state, this is given by 6 ϕ (assuming for simplicity $\omega = 0$). Under aftermarket monopoly, consumers get zero in the aftermarket. For small ϕ, the foremarket component is determined by prices (that is, product differentiation effects are of second order, as shown in the Proof of Proposition 5). Under aftermarket competition, average price in the foremarket is given by $\frac{1}{2} - 2\phi$, whereas under aftermarket monopoly we have $\frac{1}{2} - 10\phi$. This implies that the gain in foremarket consumer welfare from aftermarket power, 8 ϕ, more than compensates the loss in aftermarket consumer welfare from aftermarket power, 6 ϕ.

To understand this result, it helps to think of price competition in the foremarket as an auction, the object on the block being the newborn consumer's business. Suppose both firms have the same installed base. The difference between winning and losing the auction is the difference between becoming a large firm and becoming a small firm.[17] Aftermarket power increases the value of being a large firm; however, as shown in Proposition 4 it decreases the value of a small firm. The equilibrium value of a symmetric auction is equal to the value of the loser. So, Proposition 4 implies that the equilibrium value decreases with aftermarket power. This implies a lower price in the foremarket. For a small value of ϕ, most of the decrease in firm value corresponds to a transfer to consumers. In fact, as shown in the Proof of Proposition 5, the first-order effect of aftermarket power on social welfare is zero.

A different way of understanding this result is to divide the total effect of an increase in market power into two components: the direct effect and the strategic effect. The direct effect corresponds to assuming that prices in the foremarket are kept constant. The direct effect is then positive: greater power in the aftermarket means greater firm profits and greater firm value. The strategic effect corresponds to the change in foremarket prices resulting from the increase in aftermarket power. Under constant returns to scale, the strategic effect is negative and exactly compensates the direct effect, so that firm value is invariant with respect to the degree of market power (cf Proposition 1). Under increasing returns to scale, however, the strategic effect more than outweighs the direct effect, so that an increase in market power decreases firm value, to the benefit of consumers.[18]

Proposition 6 is restricted to the case when ϕ is small and δ is high. Fig. 1 depicts the difference in consumer welfare between the extreme cases of monopolized aftermarket and competitive aftermarket. If $i = 0$, then consumer welfare is invariant with respect to aftermarket conditions, regardless of the value of δ. This corresponds to Proposition 1 and results from my assumption of L-shaped demand curves in the aftermarket. As I mentioned earlier, there are reasons to believe consumer and social welfare to be increasing or decreasing in the degree of market power. I purposely make the assumption that, when $\phi = 0$, consumer and social welfare are invariant with respect to aftermarket conditions, so that departures from zero can be attributed to $\phi \neq 0$.

Fig. 1 also plots the difference between the M and C cases when $\phi > 0$. As can be seen, consumers are better off with monopoly aftermarket if the discount factor is sufficiently close to 1. In other words, the restriction that $\delta > delta'$ in Proposition 6 is “tight”. By contrast, the restriction that ϕ (the degree of increasing returns to scale), which I impose as a means to linearize the model by Taylor expansion, seems not to be necessary for the result that consumers benefit from aftermarket power.[19]

7. Discussion and final remarks

In this concluding section, I first summarize the results. Second, I discuss how robust the results are to some of my modeling assumptions. Next, I compare my infinite-period model to a more conventional (in the aftermarkets literature) two-period model. Finally, I briefly discuss empirical implications and evidence.

7.1. Summary of results

Previous economic literature suggests that foremarket competition partly compensates for aftermarket power. Some authors claim that consumers are considerably worse off when firms have aftermarket power, whereas other authors suggest consumers are nearly indifferent with respect to aftermarket conditions. In this paper I argue that, in the presence of increasing returns to scale in the aftermarket, consumers may actually be better off with a greater degree of aftermarket power. The idea is that the lure of future profits that increase more than proportionally with installed base size makes firms so much more aggressive that lower prices in the foremarket more than compensate for higher prices in the aftermarket.

[17] To be more precise, winning the “auction” implies becoming a large firm with probability 50% and a medium-sized firm with probability 50%; and losing the “auction” implies becoming a small firm with probability 50% and a medium-sized firm with probability 50%.

[18] Cabral and Villas-Boas (2005) refer to this situation as a *Bertrand supertrap*.

[19] I computed a fine grid of values of ϕ from 0 to $\frac{1}{6}$, the value at which a corner solution is obtained.

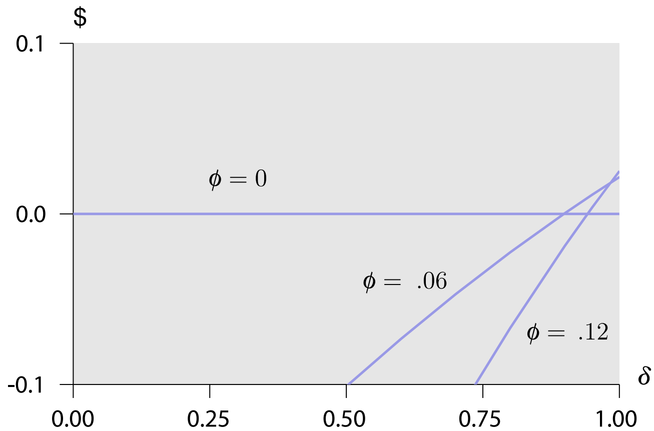

Fig. 1. Difference in consumer welfare between M and C cases as a function of δ and ϕ.

More important, my analysis also shows that increasing returns to scale imply non-trivial market share dynamics: Large firms tend to attract new consumers with higher probability than small firms; and moreover this increasing dominance effect is stronger the greater the degree of aftermarket power. This in turn implies that aftermarket power leads to more concentrated long-run market shares and a lower value of small firms.

7.2. Robustness

Propositions 2 through 6 correspond to strict inequalities. This implies that the results are not knife-edged: slightly perturbing the model does not change the sign of the main effects. This is important because, for the sake of exposition, I made a number of simplifying assumption. In particular, I assumed that changes in aftermarket power correspond to pure transfers from consumers to firms. More generally we would expect aftermarket power to imply some inefficiencies in the aftermarket (Harberger triangles). To the extent that demand elasticities are not very great, I would expect these inefficiencies to be of second order with respect to the gains implied by Propositions 5 and 6.

As mentioned earlier, the assumptions regarding the number of consumers (three in each period) and distribution of preferences (uniform) are not essential for the results. One assumption, however, does play an important role, namely the assumption that consumers have no outside option. In this sense, my results parallel the debate regarding persistence of dominance in oligopoly. Gilbert et al. (1982) argued that whenever an innovation is available an incumbent monopolist is willing to pay more for it than a potential entrant. Effectively, the incumbent has more to lose from not securing this innovation than the entrant has to gain from obtaining it. The "efficiency effect" characterized by Gilbert et al. (1982) depends crucially on the assumption that were the incumbent not to obtain the patent then the potential entrant would — and would use it to compete against the incumbent. This assumption effectively corresponds to my assumption of no outside option: if a large firm does not attract a newborn consumer, then the entrant will. Absent the no-outside-option assumption, it is no longer necessarily true that the large firm has more to gain from attracting the newborn consumer. The same is true in Gilbert et al.'s (1982) context of an incumbent monopolist: if the alternative of not acquiring an innovation is that no one will do so (with some probability), then the incumbent's innovation motives are greatly diminished, possibly below those of a potential entrant (Arrow, 1962; Reinganum, 1982).

Finally, as mentioned in Section 2, I implicitly assume consumers have a switching cost which alternates between infinite and zero. I believe my results are robust with respect to small perturbations in the values of switching cost. A more realistic model would consider a less-extreme distribution of switching costs but would most likely be intractable.

7.3. Infinite-period models and two-period models

From a methodological point of view, my approach is considerably different from the previous literature on aftermarket power. The latter is typically based on two-period models. By contrast, I build a dynamic, infinite-period model, and solve for the symmetric Markov equilibrium. It is reasonable to ask whether such a modeling investment is worthwhile. By means of a simple example, I now argue that, compared to my model, the "natural" corresponding two-period model can lead to very different results — in fact, the *opposite* result.

Suppose aftermarket value (profits plus consumer surplus) is infinitesimally decreasing in market power: $V_i^M = V_i^C - \epsilon_i$, where ϵ_i is a series of infinitesimal numbers. As shown in Section 6, the dynamic model implies that steady-state welfare is strictly greater under monopolized aftermarket even though, *at each state*, welfare is lower under monopolized aftermarket. Consider now the two-period model that best corresponds to the dynamic model's basic conditions: (a) in the first period, two firms simultaneously set prices and then three consumers simultaneously choose one of the firms; (b) in the second period, aftermarket profits are received. By symmetry, the probability that each firm attracts a consumer in the first stage is 1/2, *regardless of the degree of aftermarket power*. This implies that the weights on each possible market outcome (i.e., number of consumers per firm) are independent of aftermarket power. Since welfare is strictly lower under aftermarket power, it follows that welfare is lower overall under aftermarket power. By contrast, the dynamic model shows that, in the steady-state, welfare is greater under aftermarket power. What is missing in the two-period model is precisely the effect of aftermarket power on installed-base dynamics, that is, the fact that aftermarket power ultimately leads to a more asymmetric market structure.

7.4. Empirical evidence

As the examples in the introduction suggest, the issue of aftermarket power is of great practical relevance. Moreover, the basic structure of my model — in particular the overlapping nature of consumer "lives" — seems fairly realistic. Are the model results realistic as well?

One potentially problematic point is that Proposition 6 suggests that firms are better off when aftermarket power is shut down. In other words, if we were to consider a model with endogenous choice of aftermarket power, Proposition 6 would suggest that firms might choose a competitive aftermarket. However, in practice we observe firms pursuing aftermarket power. For example, in March 2010 Sara Lee launched a line of Nespresso-compatible coffee capsules; and soon after Nestlé sued Sara Lee in France for patent violation.

Although the purpose of this paper is not to analyze the endogenous choice of aftermarket power, I can address the above criticism in two ways. First, Proposition 6 only implies that if firms can *jointly* achieve a *long-term commitment* to a competitive aftermarket then they will do so. But this is only one of several possibilities, as Table 3 shows: that is, Proposition 6 implies the top left cell in Table 3 is a C. In addition to long-term commitment, we may also consider short-term commitment (firms can commit to aftermarket conditions in the current period) and no commitment at all. And in terms of decision-making we may consider joint decisions taken by both firms or unilateral decisions. If firms have no ability to commit at all, then they maximize profits in the aftermarket, leading to the aftermarket-power extreme

Table 3
Endogenous choice of aftermarket conditions.

	Commitment to aftermarket conditions		
	Long-term	Short-term	None
Joint decision	C	?	M
Unilateral decision	?	?	M

considered above (M). This leaves three possible cells in Table 3, where it is not obvious what the endogenous choice of aftermarket power would be. Whatever it is, the result is not inconsistent with Proposition 6.

Second, Proposition 6 depends importantly on my assumption of L-shaped demands, that is, the assumption that total value in the aftermarket is constant. (I made this assumption so as to isolate the effect of aftermarket power and increasing returns.) In this context, higher consumer welfare implies lower firm profits. With more general aftermarket demand curves, the trade-off is no longer dollar-for-dollar. For example, evidence from mobile telecommunications in Europe (Genakos and Valletti, 2011) suggests that "waterbed" effects — the increase in some prices following the decrease of other prices — is positive but not dollar-for-dollar.

Appendix A

Proof of Proposition 1. Since $\lambda_i = \lambda$, we have $u_i = u$. From Eq. (2), this implies

$$q_i = \frac{1}{2} - \left(p_i - p_j\right). \tag{9}$$

Taking the difference of Eq. (5) for $i = 2$ and $i = 0$, we have

$$p_2 - p_0 = q_2 - q_0 - (\pi_3 - \pi_2) + (\pi_1 - \pi_0) - \delta\ \frac{2}{3}(v_2 + v_0 - 2\ v_1).$$

Substituting Eq. (9) for q_i, Eq. (8) for π_i, and Eq. (6) for v_i, and simplifying, we get

$$p_2 - p_0 = -2\ (p_2 - p_0) - \frac{4\ \delta}{3 - \delta}\ (p_2 - p_0)^2.$$

The only solution such that $|p_2 - p_0| < \frac{1}{2}$ is $p_2 = p_0$. It follows that $q_2 = q_0 = \frac{1}{2}$. Moreover, by symmetry $q_1 = \frac{1}{2}$. Substituting Eq. (5) and simplifying we get

$$p_i = p = \frac{1}{2} - \frac{3}{3 - 2\ \delta}\ \pi.$$

Each consumer's discounted utility from joining the network is given by

$$u = \lambda + \frac{2}{3}\ \delta\ u$$

where $\frac{2}{3}$ is the probability the consumer survives into the next period. Solving for u we get

$$u = \frac{\lambda}{1 - \frac{2}{3}\ \delta}.$$

It follows that a consumer's net utility is given by

$$\frac{\lambda}{1 - \frac{2}{3}\ \delta} - \frac{1}{2} + \frac{3}{3 - 2\ \delta}\ \pi = -\frac{1}{2} + \frac{3}{3 - 2\ \delta}\ (\pi + \lambda).$$

It follows that any shift between π and λ that keeps the sum constant has no effect on consumer surplus. ■

Proof of Proposition 2. *Define $q \equiv q_2$. This value of q summarizes the equilibrium, since $q_0 = 1 - q_2 = 1 - q$, and $q_1 = \frac{1}{2}$ by symmetry. The proof is divided into three steps. First I solve for Case C (competitive aftermarkets). Next I solve for Case M (monopolized aftermarkets). Finally, I compare the values of q in each case. For simplicity, I will assume $\omega = 0$. Since a positive value changes utilities uniformly across states, it has no impact on the variables of interest.

□ **Case C.** $\lambda_i = i\ \phi$, $\pi_i = 0$. Substituting Eq. (3) for u_i and simplifying, we get

$$u_3 - u_1 = \frac{12\ \phi}{6 - (1 + 2\ q)\ \delta}. \tag{10}$$

Substituting Eq. (5) for p_i, q for q_2, $1 - q$ for q_0, and 0 for π_i, and simplifying, we get

$$p_2 - p_0 = 2\ q - 1 - \delta\ \frac{2}{3}(v_2 + v_0 - 2\ v_1).$$

Substituting Eq. (6) for v_i, and simplifying, the above equation implies

$$p_2 - p_0 = 2\ q - 1 - \frac{\delta}{3 - \delta}\ \left(1 - 4\ q + 4\ q^2\right).$$

From Eq. (2), we know that

$$(p_2 - p_0) - (u_3 - u_1) = \frac{1}{2} - q. \tag{11}$$

It follows that, by subtracting Eq. (10) from Eq. (11), we get

$$\frac{1}{2} - q = 2\ q - 1 - \frac{\delta}{3 - \delta}\ \left(1 - 4\ q + 4\ q^2\right) - \frac{12\ \phi}{6 - (1 + 2\ q)\ \delta}. \tag{12}$$

Let ϕ^C be the (unique) solution of Eq. (12) with respect to ϕ. Computation establishes that

$$\frac{\partial \phi^C}{\partial q} = \frac{(1 - \delta\ q)\ (27 - 12\ \delta\ q) + \delta\ (3 - \delta)\ (1 - q) + \delta^2\ q}{6\ (3 - \delta)} \tag{13}$$

which, considering that $\delta \in (0, 1)$ and $q \in [0, 1]$, is positive. This implies that, in the relevant range of values of δ and q, the relation between ϕ and q is one-to-one. Hence, there exists a unique value of q implicitly given by Eq. (12).

□ **Case M.** $\pi_i = i^2\ \phi$, $\lambda_i = 0$. In this case, we clearly have $u_3 - u_1 = 0$. (A consumer expects a payoff of λ each period it is still alive, independently of the size of the installed base.) Substituting Eq. (5) for p_i, Eq. (6) for v_i, q for q_2, $1 - q$ for q_0, $i^2\ \phi$ for π_i, and simplifying, we get

$$p_2 - p_0 = 2\ q - 1 - \frac{12\ \phi}{3 - \delta} - \frac{\delta}{3 - \delta}\ \left(1 - 4\ q + 4\ q^2\right).$$

Substituting Eq. (11) for $p_2 - p_0$, and noting that $u_3 - u_1 = 0$, we get

$$\frac{1}{2} - q = 2\ q - 1 - \frac{\delta}{3 - \delta}\ \left(1 - 4\ q + 4\ q^2\right) - \frac{12\ \phi}{3 - \delta}. \tag{14}$$

Let ϕ^M be the (unique) solution of Eq. (14) with respect to $\boldsymbol{\phi}$. Computation establishes that

$$\frac{\partial \boldsymbol{\phi}^M}{\partial q} = \frac{1}{12}(9 + \delta - 8\ \delta\ q) \tag{15}$$

which, considering that $\delta \in (0, 1)$ and $q \in [0, 1]$, is positive. This implies that, in the relevant range of values of δ and q, the relation between ϕ and q is one-to-one. Hence, there exists a unique value of q implicitly given by Eq. (14).

□ **Relation between q^M and q^C.** The last step in the proof consists of comparing the equilibrium values of q in cases M and C, which I denote by q^M and q^C, respectively. Both q^M and q^C are strictly increasing in ϕ.

Moreover, $\phi = 0$ implies that $q^M = q^C = \frac{1}{2}$ (by symmetry). It follows that $q^M > q^C$ if and only if $\Phi \equiv \phi^M - \phi^C > 0$. Solving $\frac{\partial\Phi}{\partial\delta} = 0$ with respect to q yields roots $\frac{1}{2}$ and

$$\frac{45-6\ \delta+\delta^2 \pm \sqrt{2025-1404\ \delta+270\ \delta^2-12\ \delta^3+\delta^4}}{2\ \delta\ (24-4\ \delta)}.$$

Considering that $\delta \in (0, 1)$, the latter two roots are greater than one or less than zero. It follows that, for $q \in (\frac{1}{2}, 1)$, the sign of $\frac{\partial\Phi}{\partial\delta}$ is the same as when $q = 1$. Computation establishes that

$$\left.\frac{\partial\Phi}{\partial\delta}\right|_{q=1} = -\frac{36-30\ \delta+5\ \delta^2}{12\ (3-\delta)^2}$$

which is negative, given that $\delta \in (0, 1)$. We conclude that a sufficient condition for $\Phi > 0$ when $\delta \in (0, 1)$ is that $\Phi\mid_{\delta=1} > 0$. In fact, computation establishes that

$$\Phi\mid_{\delta=1} = 1/12\ (2\ q-1)\ (2-q)\ (3-2\ q)$$

which is positive for all $q \in (\frac{1}{2}, 1)$. ■

Proof of Proposition 3. Let $M = m_{ik}$ be the Markov transition matrix across states $i, k = 0, 1, 2$. Let $[\mu_0, \mu_1, \mu_2]$ be the stationary distribution over states. Define $\mu = \mu_1$ and $q = q_2$. I next derive μ as a function of q. The first column of the Markov transition matrix is given by

$$m_{00} = q + \frac{1}{3}\ (1-q)$$
$$m_{10} = \frac{1}{6}$$
$$m_{20} = 0.$$

By definition of stationary state

$$\mu_0 = \sum_{k=0}^{2} m_{k0}\ \mu_k.$$

Symmetry implies that $\mu_0 = \mu_2 = \frac{1}{2}\ (1-\mu)$. Substituting in the above expression, we have

$$\frac{1}{2}\ (1-\mu) = \frac{1}{2}\ (1-\mu)\left(q + \frac{1}{3}\ (1-q)\right) + \frac{1}{6}\ \mu.$$

Solving for μ, we get

$$\mu = \frac{2-2\ q}{3-2\ q}. \tag{16}$$

Straightforward derivation shows that μ is decreasing in q. The result then follows from Proposition 2. ■

Proof of Proposition 4. Solving Eq. (6) for $i = 0$, we get

$$v_0 = \frac{(1-q)^2}{1-\delta}.$$

The result then follows from Proposition 2. ■

Proof of Proposition 5. Social welfare is given by two components: aftermarket total surplus and foremarket total surplus. In terms of aftermarket surplus, we have two possibilities. Either we are in more asymmetric split of installed bases ($i = 0, j = 3$ or $i = 3, j = 0$); or we are in a more symmetric split ($i = 1, j = 2$ or $i = 2, j = 1$). In the first case, total surplus is given by $3\ \omega + 0\ \phi + 3^2\ \phi = 3\ \omega + 9\ \phi$. In the second case, we have $3\ \omega + 1\ \phi + 2^2\ \phi = 3\ \omega + 5\ \phi$. So, the greater the asymmetry of installed bases, the greater is the social welfare.

The steady state probability of a more asymmetric aftermarket split of installed bases is given by $(1 - \mu)\ q$. In words, the system must start from an asymmetric state, which happens with probability $1 - \mu$; and the large firm must make the sale, which happens with probability q. We conclude that a sufficient statistic for steady-state social welfare *in the aftermraket* is

$$3\ \omega + (1-\mu)\ q\ 9\ \phi + (1-(1-\mu)\ q)5\ \phi.$$

The second component of social welfare is total surplus in the foremarket. A sufficient statistic for this surplus is total transportation costs (or the negative of). Modulo a constant term, this is given by the extra transportation cost due to firms setting different prices. Specifically, at stage $i = 0$ or $i = 2$ we must take into account consumers with addresses between 0 and $q - \frac{1}{2}$, who now purchase from a firm that's located farther away. If $p_i = p_j$, these consumers would pay a transportation cost of $\frac{1}{2} - x$, where x is their address. Now they pay a transportation cost of $\frac{1}{2} + x$. The total increase in transportation costs is given by

$$\int_0^{q-\frac{1}{2}} 2\ x\ dx = (q-1/2)^2.$$

This cost is incurred with probability $1 - \mu$, which in the steady state is equal to $\frac{q}{3-2\ q}$.

Pulling the two components together, substituting Eq. (16) for μ, and simplifying, we have the following sufficient statistic of social welfare:

$$S = \frac{15-6\ q}{3-2\ q}\ \phi + \frac{q}{3-2\ q}(q-1/2)^2.$$

Straightforward computation shows that the various terms in S are strictly increasing in q. Proposition 2 implies that $q^M > q^C$. The result follows. ■

Proof of Proposition 6. Let μ_i be the probability that, in the steady state, the system is at i. Let $\mu_1 = \mu$. By symmetry, $\mu_0 = \mu_2 = (1 - \mu)/2$ and so $\mu_0 + \mu_2 = 1 - \mu$. In terms of aftermarket states, we have the following possibilities: with probability $(\mu_0 + \mu_2)\ q_2$, all consumers are in the same installed base; otherwise, there is a split, with two consumers with one installed base and one with the other installed base.

In terms of the price paid by the newborn consumer, we have the following possibilities: with probability $(\mu_0 + \mu_2)\ q_2$, the consumer pays p_2; with probability $(\mu_0 + \mu_2)\ q_0$, the consumer pays p_0; and with probability μ_1 the consumer pays p_1. Defining $q_2 = q$, we can compute consumer welfare in the steady state as follows:

$$\begin{aligned} C = {} & ((1-\mu)\ q)(3\ \lambda_3) + (\mu + (1-\mu)\ q)(\lambda_1 + 2\ \lambda_2) - \\ & -(1-\mu)\ (1-q)\ p_0 - \mu\ p_1 - (1-\mu)\ q\ p_2. \end{aligned} \tag{17}$$

Note that, at $\phi = 0, p_i = q = \mu = \frac{1}{2}$ and $\lambda_i = 0$. Moreover, substituting $q = \mu = \frac{1}{2}$ in Eq. (5) implies

$$p_0 + 2\ p_1 + p_2 = 2 - (\pi_3 + \pi_2 - \pi_1 - \pi_0) - \delta\ \frac{4}{3}\ (v_2 - v_0).$$

Using this, I can differentiate Eq. (17) to get

$$\begin{aligned} \left.\frac{dC}{d\phi}\right|_{\phi=0} = {} & \frac{3}{4}\frac{d}{d\phi}(\lambda_1 + 2\lambda_2 + \lambda_3) + \frac{1}{4}\frac{d}{d\phi}(\pi_3 + \pi_2 - \pi_1 - \pi_0) + \\ & + \frac{1}{3}\delta\frac{d}{d\phi}(v_2 - v_0). \end{aligned} \tag{18}$$

□ **Consider first Case C.** $\lambda_i = i\,\phi$, $\pi_i = 0$. Substituting $i\,\phi$ for λ_i and simplifying, we get

$$\frac{3}{4}\;\frac{d}{d\phi}(\lambda_1 + 2\,\lambda_2 + \lambda_3) = 6.$$

Moreover,

$$\frac{d(v_2 - v_0)}{d\phi}\bigg|_{\phi=0} = \frac{\partial(v_2 - v_0)}{\partial q}\;\frac{dq}{d\phi}\bigg|_{q=\frac{1}{2}}.$$

From Eq. (6), I determine that

$$\frac{\partial(v_2 - v_0)}{\partial q}\bigg|_{q=\frac{1}{2}} = \frac{3\,(12-4\,\delta)}{2\,(9-9\,\delta+2\,\delta^2)}.$$

Differentiating Eq. (13) with respect to q, substituting $q = \frac{1}{2}$, and inverting, yields

$$\frac{dq}{d\phi}\bigg|_{q=\frac{1}{2}} = \frac{2}{3-\delta}.$$

Substituting all of these expressions in Eq. (18), I finally get

$$\frac{dC}{d\phi} = \frac{2\left(27-25\,\delta+6\,\delta^2\right)}{(3-\delta)\,(3-2\,\delta)}. \tag{19}$$

Notice that

$$\lim_{\delta\to 1}\;\frac{dC}{d\phi} = 8. \tag{20}$$

□ **Consider first Case M.** $\pi_i = i^2\,\phi$, $\lambda_i = 0$. Substituting $i^2\,\phi$ for π_i, we get

$$\frac{1}{4}\;\frac{d}{d\phi}(\pi_3 + \pi_2 - \pi_1 - \pi_0) = 3.$$

Regarding the second row in Eq. (18), I now must consider the fact that v_i depends on π_i, and so when computing the derivative with respect to ϕ, I must consider both the direct partial and the effect of ϕ through changes in q. The two partial derivatives are given by

$$\frac{\partial(v_2 - v_0)}{\partial\phi} = \frac{24-12\,\delta}{2\,(9-9\,\delta+2\,\delta^2)}$$
$$\frac{\partial(v_2 - v_0)}{\partial q} = \frac{12-8\,\delta\,q}{2\,(9-9\,\delta+2\,\delta^2)}.$$

Differentiating Eq. (15) with respect to q, substituting $q = \frac{1}{2}$, and inverting, yields

$$\frac{dq}{d\phi}\bigg|_{q=\frac{1}{2}} = \frac{4}{3-\delta}.$$

Substituting all of these expressions in Eq. (18), I finally get

$$\frac{dC}{d\phi} = \frac{27-7\,\delta}{(3-\delta)\,(3-2\,\delta)}. \tag{21}$$

Notice that

$$\lim_{\delta\to 1}\;\frac{dC}{d\phi} = 10. \tag{22}$$

Comparing Eqs. (20) and (22), the result follows. ■

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